Worksheet for October 24

Problems marked with an asterisk are to be placed in your math diary.

(1.*) Consider the triple integral $\int\int\int_B xyz\ dV$, where $B = [0,2] \times [0,2] \times [0,2]$.

(i) Calculate the simple Riemann sum $f(1,1,1) \cdot \mathrm{vol}(B)$. Note this is the Riemann sum obtained by not partitioning B and choosing the midpoint of each factor $[0,2]$.

(ii) Divide each interval [0,2] into two subintervals of equal size, say, A_1, A_2 on the x-axis, B_1, B_2 on the y-axis and C_1, C_2 on the z-axis. This partitions B into eight small cubes, $A_i \times B_j \times C_k$, with $1 \leq i, j, k \leq 2$. Choose a point $(x_i, y_j, z_k) \in A_i \times B_j \times C_k$, where x_i is the center of A_i, y_j the center of B_j and z_k, the center of C_k. Calculate the Riemann sum

$$\Sigma_{i=1}^2 \Sigma_{j=1}^2 \Sigma_{k=1}^2 f(x_i, y_j, z_k) \cdot \mathrm{vol}(A_i \times B_j \times C_k).$$

(iii) Do you think your answers in (i) and (ii) are less than, greater than, or equal to the actual value of $\int\int\int_B xyz\ dV$?

(iv) Calculate $\int\int\int_B xyz\ dV$ using Fubini's theorem for solid rectangles and compare this with your answers in (i) and (ii).

(v) How would your answers in (i) and (ii) change if you took different partitions or different points in each partition? Confirm this with different examples.

(2.*) Let B denote the solid sphere of radius R centered at the origin.

(i) Set up the triple integral $\int\int\int_B f(x,y,z)\ dV$ in three different ways, thinking of B as a z-simple, x-simple, and y-simple region.

(ii) Make an educated guess of the value of $\int\int\int_B x\ dV$. Hint: What is the average value of x over B?

(iii) Use Fubini's theorem to calculate $\int\int\int_B x\ dV$. Was your guess in (ii) correct? Note: The actual calculation can be hard or not so hard - its up to you!